Solución de ejercicios impares sección 1.1

Ejercicio 1

a) $>$	b) $>$	c) $<$	d) $<$	e) $=$	f) $=$

Ejercicio 2

a)	$4+2[5+3(6+2)]$ $=4+2[5+3(8)]$ $=4+2[5+24]$ $=4+58$ $=62$
b)	-9
c)	$\|3+\pi\|-4\|15-2\cdot 9\|$ $=3+\pi-4\|15-2\cdot 9\|$ $=3+\pi-4\|15-18\|$ $=3+\pi-4\|-3\|$ $=3+\pi-4\cdot 3$ $=3+\pi-12$ $=\pi-9$
d)	30

e)	$-\left(2\cdot 8+16\div -2\right)+\left(3^{3}\div 5\cdot 25\right)$ $=-\left(16-8\right)+\left(27\div 5\cdot 25\right)$ $=-8+\left(\frac{27}{5}\cdot 25\right)$ $=-8+\left(27\cdot 5\right)$ $=127$
f)	-205
g)	$\frac{2}{5}+\left\{2-\frac{14}{107}\left[\frac{6}{4}+\frac{1}{2}-5\left(\frac{5}{2}-\frac{4}{7}\right)\right]\right\}$ $=\frac{2}{5}+\left\{2-\frac{14}{107}\left[\frac{6}{4}+\frac{1}{2}-5\left(\frac{7\cdot 5-2\cdot 4}{14}\right)\right]\right\}$ $=\frac{2}{5}+\left\{2-\frac{14}{107}\left[\frac{6}{4}+\frac{1}{2}-5\left(\frac{27}{14}\right)\right]\right\}$ $=\frac{2}{5}+\left\{2-\frac{14}{107}\left[\frac{6}{4}+\frac{1}{2}-\frac{135}{14}\right]\right\}$ $=\frac{2}{5}+\left\{2-\frac{14}{107}\left[\frac{7\cdot 6+14\cdot 1-2\cdot 135}{28}\right]\right\}$ $=\frac{2}{5}+\left\{2-\frac{14}{107}\left[\frac{-214}{28}\right]\right\}$ $=\frac{2}{5}+\left\{2+1\right\}$ $=\frac{2+3\cdot 5}{5}$ $=\frac{17}{5}$
h)	$\frac{6}{5}$

i)	$\left\|6+\left(\frac{2}{3}\right)^{-2}\right\|+\frac{4}{7}\left(\frac{6}{5}-\frac{42}{35}\right)$ $=\left\|6+\left(\frac{3}{2}\right)^{2}\right\|+\frac{4}{7}\left(\frac{6}{5}-\frac{6}{5}\right)$ $=\left\|6+\frac{9}{4}\right\|+\frac{4}{7}\cdot 0$ $=\left\|\frac{6\cdot 4+9}{4}\right\|$ $=\frac{33}{4}$
j)	$\frac{207}{2}$
k)	$\frac{2}{7}\left[3\left(\frac{5}{4}+\frac{36}{5}-\frac{129}{20}\right)-2\cdot\frac{3}{5}\div\frac{-1}{25}\right]$ $=\frac{2}{7}\left[3\left(\frac{5\cdot 5+4\cdot 36-129}{20}\right)-\frac{6}{5}\div\frac{-1}{25}\right]$ $=\frac{2}{7}\left[3\cdot 2-\frac{6}{5}\cdot\frac{-25}{1}\right]$ $=\frac{2}{7}[3\cdot 2+30]$ $=\frac{2}{7}\cdot 36$ $=\frac{72}{7}$
l)	$\frac{2}{17}$

m)	$\dfrac{2-\left(4\cdot 3^{-2}-2\cdot 3^{-2}\right)}{\left(3^{-1}\cdot 2\right)^{2}-\left(-1\right)^{3}}$ $=\dfrac{2-\left(4\cdot\dfrac{1}{3^2}-2\cdot\dfrac{1}{3^2}\right)}{\left(\dfrac{1}{3}\cdot 2\right)^{2}-\left(-1\right)^{3}}$ $=\dfrac{2-\left(4\cdot\dfrac{1}{9}-2\cdot\dfrac{1}{9}\right)}{\left(\dfrac{2}{3}\right)^{2}--1}$ $=\dfrac{2-\left(\dfrac{4}{9}-\dfrac{2}{9}\right)}{\dfrac{4}{9}+1}$ $=\dfrac{2-\dfrac{2}{9}}{\dfrac{13}{9}}$ $=\dfrac{\dfrac{16}{9}}{\dfrac{13}{9}}$ $=\dfrac{16}{13}$
n)	-6

Ejercicio 3

a)	$\left\|3+\sqrt{125}\right\|-4\left\|15-\sqrt{20}\right\|$ $=\left\|3+\sqrt{5^2\cdot 5}\right\|-4\left\|15-\sqrt{2^2\cdot 5}\right\|$ $=\left\|3+5\sqrt{5}\right\|-4\left\|15-2\sqrt{5}\right\|$ $=3+5\sqrt{5}-4\left(15-2\sqrt{5}\right)$ $=3+5\sqrt{5}-30+8\sqrt{5}$ $=13\sqrt{5}-27$
b)	$-14\sqrt{5}$
c)	$\sqrt[3]{648}-5\sqrt[3]{686}-\sqrt[3]{16}+\sqrt[3]{3}$ $=\sqrt[3]{3^3\cdot 3\cdot 2^3}-5\sqrt[3]{7^3\cdot 2}-\sqrt[3]{2^3\cdot 2}+\sqrt[3]{3}$ $=3\cdot 2\sqrt[3]{3}-5\cdot 7\sqrt[3]{2}-2\sqrt[3]{2}+\sqrt[3]{3}$ $=6\sqrt[3]{3}-35\sqrt[3]{2}-2\sqrt[3]{2}+\sqrt[3]{3}$ $=7\sqrt[3]{3}-37\sqrt[3]{2}$
d)	$13\sqrt{2}+\dfrac{3\sqrt[3]{3}}{20}$

e)	$\left[\frac{(-2)^2}{4}-2^{-3}\div 2^{-1}\right]^{-1}-\frac{\sqrt{3}}{3}\left(2\sqrt{3}-3\sqrt{3}\right)$ $=\left[\frac{4}{4}-\frac{1}{2^3}\div\frac{1}{2}\right]^{-1}-\frac{2\sqrt{3^2}}{3}+\frac{3\sqrt{3^2}}{3}$ $=\left[1-\frac{1\cdot 2}{8\cdot 1}\right]^{-1}-\frac{2\cdot 3}{3}+3$ $=\left[1-\frac{1}{4}\right]^{-1}+1$ $=\left[\frac{4-1}{4}\right]^{-1}+1$ $=\frac{4}{3}+1$ $=\frac{7}{3}$
f)	$12\sqrt[4]{2}-2\sqrt{2}+9\sqrt{10}$

Ejercicio 4

a)	$\frac{6}{\sqrt{7}}=\frac{6}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\frac{6\sqrt{7}}{\sqrt{7^2}}=\frac{6\sqrt{7}}{7}$
b)	$\frac{\sqrt{7}}{7}$

c)	$\frac{-17}{\sqrt[3]{20}}=\frac{-17}{\sqrt[3]{2^2\cdot 5}}\cdot\frac{\sqrt[3]{2\cdot 5^2}}{\sqrt[3]{2\cdot 5^2}}=\frac{-17\sqrt[3]{50}}{\sqrt[3]{2^3\cdot 5^3}}=\frac{-17\sqrt[3]{50}}{10}$
d)	$\frac{25\sqrt[5]{4}}{2}$
e)	$\frac{14}{\sqrt[3]{12}}=\frac{14}{\sqrt[3]{2^2\cdot 3}}\cdot\frac{\sqrt[3]{2\cdot 3^2}}{\sqrt[3]{2\cdot 3^2}}=\frac{14\sqrt[3]{18}}{12}=\frac{7\sqrt[3]{18}}{3}$
f)	$\frac{\sqrt{2}}{4}$

Ejercicio 5

a)	$\frac{\sqrt{3}}{3}=\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{3}{3\sqrt{3}}=\frac{1}{\sqrt{3}}$
b)	$\frac{2}{3\sqrt[5]{4}}$
c)	$\frac{\sqrt{2}}{\sqrt{14}}=\frac{\sqrt{2}}{\sqrt{14}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2}{\sqrt{14\cdot 2}}=\frac{2}{\sqrt{7\cdot 2\cdot 2}}=\frac{2}{2\sqrt{7}}=\frac{1}{\sqrt{7}}$
d)	$\frac{1\sqrt[4]{8}}{100}$

e)	$\frac{\sqrt[3]{12}}{10}=\frac{\sqrt[3]{3\cdot 2^2}}{10}\cdot\frac{\sqrt[3]{3^2\cdot 2}}{\sqrt[3]{3^2\cdot 2}}=\frac{\sqrt[3]{3^3\cdot 2^3}}{10\sqrt[3]{3^2\cdot 2}}=\frac{3\cdot 2}{10\sqrt[3]{18}}=\frac{3}{5\sqrt[3]{18}}$
f)	$\frac{1}{5}$

Solución de ejercicios sección 1.2

Ejercicio 1

a)	$\frac{x^6y^3p^2k^3}{x^{10}y^9m^2k^{-5}}=\frac{x^{6-10}y^{3-9}p^2k^{3--5}}{m^2}=\frac{x^{-4}y^{-6}p^2k^8}{m^2}=\frac{p^2k^8}{x^4y^6m^2}$
b)	$m^{29}t^{21}p^{23}$
c)	$\frac{20a^3m^2c^{11}d^3}{5a^3mc^{10}d^2}=\frac{20}{5}a^{3-3}m^{2-1}c^{11-10}d^{3-2}=4a^0m^1c^1d^1=4mcd$
d)	$\frac{2x^4y^2m^{12}}{3n}$
e)	$\left(\frac{25a^{\frac{3}{2}}b^{\frac{1}{3}}c^{-2}}{16ab^{\frac{1}{3}}}\right)^{\frac{1}{2}}=\left(\frac{25a^{\frac{1}{2}}}{16c^2}\right)^{\frac{1}{2}}=\frac{5a^{\frac{1}{4}}}{4\lvert c\rvert}=\frac{5}{4\lvert c\rvert}\sqrt[4]{a}$
f)	$\frac{a^{\frac{17}{3}}}{4096b}$

Ejercicio 2

a)	$(6x+3)+(2x^2+8x+8)$ $=6x+3+2x^2+8x+8$ $=2x^2+14x+11$

b)	$-x^2 - x + 6$
c)	$\left(3x^4 - 4x^3 + 6x^2\right) - \left(x^5 + 3x^3 + 12x\right)$ $= 3x^4 - 4x^3 + 6x^2 - x^5 - 3x^3 - 12x$ $= -x^5 + 3x^4 - 7x^3 + 6x^2 - 12x$
d)	$5x^5 - 5x^4 + 10x^3$
e)	$\frac{3}{2}a(a-2)\left(-2a - \frac{1}{2}\right) + \frac{5}{2}$ $= \left(\frac{3}{2}a^2 - 3a\right)\left(-2a - \frac{1}{2}\right) + \frac{5}{2}$ $= -3a^3 - \frac{3}{4}a^2 + 6a^2 + \frac{3}{2}a + \frac{5}{2}$ $= -3a^3 + \frac{21}{4}a^2 + \frac{3}{2}a + \frac{5}{2}$
f)	$\frac{3x^3}{5} - \frac{x^2}{10} - \frac{7x}{2} - \frac{4}{3}$
g)	$(w+5)\left(w^2 - 5w + 25\right)$ $= w^3 - 5w^2 + 25w + 5w^2 - 25w + 125$ $= w^3 + 125$
h)	$x^2 + x - y^2 + 3y - 2$

Ejercicio 3

a)	$4x^3-8x^2-51x-45 \quad \lfloor\, 2x+3$ $-(4x^3+6x^2) \qquad 2x^2-7x-15$ $0\ -14x^2-51x$ $-(-14x^2-21x)$ $0\ -30x-45$ $-(-30x-45)$ 0 **Cociente:** $2x^2-7x-15$ **residuo: 0.**
b)	Cociente: $-x^2+x+12$ Residuo: 0
c)	$1\ \ -2\ \ -6\ \ -45\ \lfloor 6$ $\downarrow\ \ 6\ \ 24\ \ 108$ $1\ \ 4\ \ 18\ \ 63$ Cociente: $x^2+4x+18$ residuo: 63
d)	Cociente: $3x^2+5x-1$ Residuo: 1

e)	$$\begin{array}{l} 2x^4 - 3x^3 + 0x^2 + x + 1 \quad \lfloor\, x^2 + 2x + 1 \\ -\left(2x^4 + 4x^3 \;\; + 2x^2\right) \qquad 2x^2 - 7x + 12 \\ 0 \quad -7x^3 - 2x^2 \; + \; x \\ \quad -\left(-7x^3 - 14x^2 - 7x\right) \\ \qquad 0 \quad 12x^2 + \; 8x + 1 \\ \qquad\quad -\left(12x^2 + 24x + 12\right) \\ \qquad\qquad 0 \quad -16x - 11 \end{array}$$	Cociente: $2x^2 - 7x + 12$ Residuo: $-16x - 11$
f)	Cociente: $49x^2 - 203x + 86$ Residuo: $358x - 172$	
g)	$$\begin{array}{ccccc} \frac{3}{2} & -11 & 20 & 3 & \lfloor 4 \\ \downarrow & 6 & -20 & 0 & \\ \hline \frac{3}{2} & -5 & 0 & 3 & \end{array}$$	Cociente: $\left(\frac{3}{2}x^2 - 5x\right)$ residuo: 3
h)	Cociente: $2w^3 + w - 3$ Residuo: $4w - 3$	

Ejercicio 4

a)	$\left(\frac{1}{2}ab^2c^3-\frac{1}{4}\right)^2=\frac{1}{4}a^2b^4c^6-\frac{1}{4}ab^2c^3+\frac{1}{16}$
b)	$686x^4y^2-392x^3y^3+56x^2y^4$
c)	$(2a+b)^2-4(a+b)(a-b)$ $=4a^2+4ab+b^2-4\left(a^2-b^2\right)$ $=4a^2+4ab+b^2-4a^2+4b^2$ $=4ab+5b^2$
d)	$8x^3+54xy^4+27y^6+25y^2$
e)	$-3\left[1-(x+2)^2\right]+(3-2x)(2x-5)$ $=-3\left[1-\left(x^2+4x+4\right)\right]+6x-15-4x^2+10x$ $=-3\left[1-x^2-4x-4\right]+16x-15-4x^2$ $=-3\left[-x^2-4x-3\right]+16x-15-4x^2$ $=3x^2+12x+9+16x-15-4x^2$ $=-x^2+28x-6$
f)	$5x^2-38x+77$

g)	$\left(\frac{x+y}{2}\right)^2-\left(\frac{x-y}{2}\right)^2$ $=\frac{x^2}{4}+\frac{xy}{2}+\frac{y^2}{4}-\left(\frac{x^2}{4}-\frac{xy}{2}+\frac{y^2}{4}\right)$ $=\frac{x^2}{4}+\frac{xy}{2}+\frac{y^2}{4}-\frac{x^2}{4}+\frac{xy}{2}-\frac{y^2}{4}$ $=xy$
h)	$-\frac{2y^2}{x}-6x$
i)	$-(x-y)^2(x+y)^2-(x^2+y^2)(x^2-y^2)$ $=-(x-y)(x-y)(x+y)(x+y)-(x^4-y^4)$ $=-(x-y)(x+y)(x-y)(x+y)-x^4+y^4$ $=-(x^2-y^2)(x^2-y^2)-x^4+y^4$ $=-(x^4-2x^2y^2+y^4)-x^4+y^4$ $=-x^4+2x^2y^2-y^4-x^4+y^4$ $=-2x^4+2x^2y^2$
j)	$2y^2\sqrt[3]{2x}+5x^2\sqrt[3]{2y}$

k)	$7ab\sqrt[3]{3a^2}+b^2\sqrt[3]{375a^2}+2\sqrt[3]{24a^5b^3}-\sqrt[3]{81a^2b^6}$ $=7ab\sqrt[3]{3a^2}+b^2\sqrt[3]{5^3\cdot 3\cdot a^2}+2\sqrt[3]{2^3\cdot 3a^3a^2b^3}-\sqrt[3]{3^3\cdot 3a^2b^3\cdot b^3}$ $=7ab\sqrt[3]{3a^2}+5b^2\sqrt[3]{3a^2}+2\cdot 2ab\sqrt[3]{3a^2}-3b^2\sqrt[3]{3a^2}$ $=7ab\sqrt[3]{3a^2}+5b^2\sqrt[3]{3a^2}+4ab\sqrt[3]{3a^2}-3b^2\sqrt[3]{3a^2}$ $=11ab\sqrt[3]{3a^2}+2b^2\sqrt[3]{3a^2}$
l)	$x^3+\frac{121x^2}{4}+305x+1025$
m)	$\frac{\sqrt{27a^5y^4}}{\sqrt{48a^3y}}-2\sqrt[6]{\frac{729a^5y^9}{a^{-1}}}+\sqrt{\frac{64a^2y^3}{9}}$ $=\sqrt{\frac{3^2\cdot 3a^5y^4}{2^4\cdot 3a^3y}}-2\sqrt[6]{3^6a^5ay^9}+\sqrt{\frac{2^6a^2y^2y}{3^2}}$ $=\sqrt{\frac{3^2a^2y^3}{2^4}}-2\cdot 3ay\sqrt[6]{y^3}+\frac{2^3}{3}ay\sqrt{y}$ $=\frac{3}{2^2}ay\sqrt{y}-6ay\sqrt[6]{y^3}+\frac{8}{3}ay\sqrt{y}$ $=\frac{3}{4}ay\sqrt{y}-6ay\sqrt{y}+\frac{8}{3}ay\sqrt{y}$ $=-\frac{31}{12}ay\sqrt{y}$
n)	$x^3y+2y\sqrt{x}+x-3y^2$

Solución de ejercicios sección 1.3

Ejercicio 1

a)	$4x^2+12x=4x(x+3)$
b)	$3ab(b^2-3a^2)$
c)	$20abc^2+40a^2bc=20abc(c+2a)$
d)	$4mn(m+2n-3m^2n^2)$
e)	$17x^4y^4+51x^2y^2-85xy=17xy(x^3y^3+3xy-5)$
f)	$3m^2n^2(5mn-6+7m^2n^2)$
g)	$\frac{6}{5}x^2y^3z+\frac{12}{5}x^3y^3+\frac{9}{5}x^4yz^3=\frac{3}{5}x^2y(2y^2z+4xy^2+3x^2z^3)$
h)	$\frac{2}{15}a^4b(10ab^2-15a^3+9b^3)$
i)	$(a+b)(c-5)+(a-b)(c-5)=(c-5)(a+b+a-b)=2a(c-5)$
j)	$(3x-1)(x+1)$

k)	$(x+y-3)(x^2+1)-x^2-1=(x+y-3)(x^2+1)-(x^2+1)=(x^2+1)(x+y-3-1)=(x^2+1)(x+y-4)$
l)	$(x+7)(y+2)$

Ejercicio 2

a)	$16a^2-1=(4a+1)(4a-1)$
b)	$(7x-8y)(7x+8y)$
c)	$144-w^8=(12-w^4)(12+w^4)=(\sqrt[4]{12}+w)(\sqrt[4]{12}-w)(\sqrt{12}+w^2)(12+w^4)$
d)	$\left(xy^3-\frac{2}{3}\right)\left(xy^3+\frac{2}{3}\right)$
e)	$\frac{b^4}{36}-\frac{x^2}{25}=\frac{1}{900}(5b^2-6x)(5b^2+6x)$
f)	$-8(x+1)$

g)	$49x^2-(2+3x)^2$ $=(7x-(2+3x))(7x+(2+3x))$ $=(7x-2-3x)(7x+2+3x)$ $=(4x-2)(10x+2)$ $=4(2x-1)(5x+1)$
h)	$(x+4)^2$
i)	$1-14w+49w^2=(1-7w)^2$ $\downarrow \qquad \downarrow$ $1 \qquad -7w$ $1 \qquad -7w$
j)	$(x^3+3)^2$
k)	$x^2-24xy^2+144y^4=(x+12y^2)^2$ $\downarrow \qquad \downarrow$ $x \qquad 12y^2$ $x \qquad 12y^2$
l)	$\frac{1}{64}(5-2ab^2c^4)^2$

m)	$\frac{a^2b^2}{9}+2abc+9c^2=\left(\frac{ab}{3}+3c\right)^2$ $\downarrow \qquad \downarrow$ $\frac{ab}{3} \qquad 3c$ $\frac{ab}{3} \qquad 3c$
n)	$\frac{1}{16}(2x+y)^2$
o)	$m^6-8y^9=(m^2-2y^3)(m^4+2m^3y^3+4y^6)$
p)	$(1+9xy)(1-9xy+81x^2y^2)$
q)	$216x^9-64y^6z^9$ $=(6x^3-4y^2z^3)(36x^6+24x^3y^2z^3+16y^4z^6)$ $=8(3x^3-2y^2z^3)(9x^6+6x^3y^2z^3+4y^4z^6)$
r)	$4(3x^2+4)$

s)	$x^3-(x+3)^3$ $=(x-(x+3))\left(x^2+x(x+3)+(x+3)^2\right)$ $=(x-x-3)\left(x^2+x^2+3x+x^2+6x+9\right)$ $=-3\left(3x^2+9x+9\right)$ $=-9\left(x^2+3x+3\right)$
t)	$(11-4x)\left(31x^2-103x+91\right)$

Ejercicio 3

a)	$ab+cb+ad+cd=b(a+c)+d(a+c)=(b+d)(a+c)$
b)	$\left(x^2+1\right)\left(x-y^2\right)$
c)	$2a+1-6ax-3x=(2a+1)-3x(2a+1)=(2a+1)(1-3x)$
d)	$(2m-3x)(n+z-2)$
e)	$a^3b-a^2-a^2b^4+ab^3=a^2(ab-1)-ab^3(ab-1)=(ab-1)\left(a^2-ab^3\right)=a(ab-1)\left(a-b^3\right)$
f)	$(2x-y)\left(2x^2+xy^2-y\right)$

g)	$5a^3+3a^2b-20a^2-12ab+80a+48b$ $5a^3-20a^2+80a+3a^2b-12ab+48b$ $=5a\left(a^2-4a+16\right)+3b\left(a^2-4a+16\right)$ $=\left(5a+3b\right)\left(a^2-4a+16\right)$
h)	$\left(2x^2+1\right)\left(2a^2b^3-3a^2b-z^4\right)$

Ejercicio 4

a)	$x^2+x-12=\left(x+4\right)\left(x-3\right)$ $\downarrow \quad \downarrow$ $x \quad 4$ $x \quad -3$
b)	$\left(a-2\right)\left(a+8\right)$
c)	$-\left(2x^2+x-3\right)=-\left(2x+3\right)\left(x-1\right)$ $\downarrow \quad \downarrow$ $2x \quad 3$ $x \quad -1$
d)	$\left(x+9\right)\left(x+15\right)$

e)	$x^2-10x+25=(x-5)^2$ $\downarrow \quad\quad \downarrow$ $x \quad\quad -5$ $x \quad\quad -5$
f)	$(m-20)(m+16)$
g)	$14x^2+x-3=(7x-3)(2x+1)$ $\downarrow \quad\quad \downarrow$ $7x \quad\quad -3$ $2x \quad\quad 1$
h)	$-(x-3)(5x-3)$
i)	$x^2-8x-105=(x-15)(x+7)$ $\Delta=(-8)^2-4\cdot1\cdot-105=484$ $x_1=\frac{--8+\sqrt{484}}{2\cdot1}=15$ $x_2=\frac{--8-\sqrt{484}}{2\cdot1}=-7$
j)	$(2x-3)^2$

Ejercicio 5

a)	m^4+64 $=m^4+16m^2+64-16m^2$ $=\left(m^2+8\right)^2-16m^2$ $=\left(\left(m^2+8\right)-4m\right)\left(\left(m^2+8\right)+4m\right)$ $=\left(m^2-4m+8\right)\left(m^2+4m+8\right)$
b)	$-\left(-5x^2+\sqrt{10}x-1\right)\left(5x^2+\sqrt{10}x+1\right)$
c)	m^8+324 $=\left(m^8+36m^4+324\right)-36m^4$ $=\left(\left(m^4+18\right)^2-6m^2\right)\left(\left(m^4+18\right)^2+6m^2\right)$ $=\left(m^4-6m^2+18\right)\left(m^4+6m^2+18\right)$
d)	$4\left(m^4n^2-2m^2nxy+2x^2y^2\right)\left(m^4n^2+2m^2nxy+2x^2y^2\right)$
e)	$4x^4-4x^2$ $=\left(4x^4-4x^2+1\right)-1$ $=\left(\left(2x^2-1\right)-1\right)\left(\left(2x^2-1\right)+1\right)$ $=2x^2\left(2x^2-2\right)$ $=4x^2\left(x^2-1\right)$ $=4x^2\left(x-1\right)\left(x+1\right)$
f)	$\left(x+5\right)\left(x+1\right)$

g)	$x^2-6x+9-40-9$ $=\left(x^2-6x+9\right)-49$ $=\left(x-3\right)^2-49$ $=\left(\left(x-3\right)-7\right)\left(\left(x-3\right)+7\right)$ $=\left(x-10\right)\left(x+4\right)$
h)	$\left(m^2-2m+4\right)\left(m^2+2m+4\right)$
i)	x^2-xy $=\left(x^2-xy+\frac{1}{4}y^2\right)-\frac{1}{4}y^2$ $=\left(x-\frac{1}{2}y\right)^2-\frac{1}{4}y^2$ $=\left(x-\frac{1}{2}y+\frac{1}{2}y\right)\left(x-\frac{1}{2}y-\frac{1}{2}y\right)$ $=x\left(x-y\right)$
j)	$\frac{1}{4}\left(2x+y\right)\left(2x-3y\right)$

Ejercicio 6

a)	$\text{si } u=x^2$ x^4-7x^2-30 $=u^2-7u-30$ $=\left(u-10\right)\left(u+3\right)$ $=\left(x^2-10\right)\left(x^2+3\right)$ $=\left(x+\sqrt{10}\right)\left(x-\sqrt{10}\right)\left(x^2+3\right)$

b)	$\left(y^2+7\right)\left(y^2+3\right)$
c)	si $u=(2-x)^2$ $(2-x)^4-(2-x)^2-6$ $=u^2-u-6$ $=(u-3)(u+2)$ $=\left((2-x)^2-3\right)\left((2-x)^2+2\right)$ $=\left(1-4x+x^2\right)\left(6-4x+x^2\right)$
d)	$\left(a^2+4\right)(a-3)(a+3)$
e)	si $u=x^2$ $\frac{x^4}{4}-\frac{6}{11}x^2+\frac{36}{121}$ $=\frac{u^2}{4}-\frac{6u}{11}+\frac{36}{121}$ $=\frac{1}{484}\left(121u^2-264u+144\right)$ $=\frac{1}{484}(11u-12)^2$ $=\frac{1}{484}\left(11x^2-12\right)^2$
f)	$(2x+11)^2$

g)	si $u = a-2$ $4(a-2)^2 - 8(a-2) + 4$ $= 4u^2 - 8u + 4$ $= 4(u^2 - 2u + 1)$ $= 4(u-1)^2$ $= 4(a-2-1)^2$ $= 4(a-3)^2$
h)	$(x+5)(x-6)$

Ejercicio 7

a)	$4x^4 - 4x = 4x(x^3-1) = 4x(x-1)(x^2+x+1)$
b)	$y^2(y+6)(y+5)$
c)	$h^{12} - b^{12}$ $= (h^6 - b^6)(h^6 + b^6)$ $= (h^3 - b^3)(h^3 + b^3)(h^6 + b^6)$ $= (h-b)(h^2 + hb + b^2)(h+b)(h^2 - hb + b^2)(h^2 + b^2)(h^4 - h^2b^2 + b^4)$
d)	$x(2x^2 - 3y)(4x^4 + 6x^2y + 9y^2)(2x^2 + 3y)(4x^4 - 6x^2y + 9y^2)$

e)	$9x^2 - y^2 + 24x + 16$ $= (9x^2 + 24x + 16) - y^2$ $= (3x+4)^2 - y^2$ $= (3x+4-y)(3x+4+y)$
f)	$-(ab + a - 9b)(ab - a + 9b)$
g)	$-x^4 - x^3 + 8x^2 + 12x = -x(x^3 + x^2 - 8x - 12)$ Utilizando división sintética, se tiene $\begin{array}{cccc|c} 1 & 1 & -8 & -12 & -2 \\ \downarrow & -2 & 2 & 12 & \\ \hline 1 & -1 & -6 & 0 & \end{array}$ Así $-x^4 - x^3 + 8x^2 + 12x$ $= -x(x^3 + x^2 - 8x - 12)$ $= x(-x^2 + x + 6)(x+2)$ $= -x(x^2 - x - 6)(x+2)$ $= -x(x-3)(x+2)^2$
h)	$x^2(x+1)(x+2)(x+3)$

i)	$4(m+5)^3(m-1)^2+2(m+5)^4(m-1)$ $=2(m+5)^3(m-1)[2(m-1)+(m+5)]$ $=2(m+5)^3(m-1)[2m-2+m+5]$ $=2(m+5)^3(m-1)[3m+3]$ $=6(m+5)^3(m-1)(m+1)$

Solución a los ejercicios de la sección 1.4

Ejercicio 1

a)	$\frac{x^6-1}{x^3-x}=\frac{(x^3+1)(x^3-1)}{x(x^2-1)}=\frac{(x+1)(x^2-x+1)(x-1)(x^2+x+1)}{x(x+1)(x-1)}=\frac{(x^2-x+1)(x^2+x+1)}{x}$
b)	$\frac{x-3}{x+3}$
c)	$\frac{x^4+3x^3+2x^2}{x^3+2x^2+x}=\frac{x^2(x^2+3x+2)}{x(x^2+2x+1)}=\frac{x^2(x+1)(x+2)}{x(x+1)^2}=\frac{x(x+2)}{x+1}$
d)	$\frac{x-1}{x-8}$
e)	$\frac{m^2-16}{m^2-8(m-2)}=\frac{m^2-16}{m^2-8m+16}=\frac{(m-4)(m+4)}{(m-4)(m-4)}=\frac{m+4}{m-4}$
f)	$\frac{x(x-1)}{x-6}$
g)	$\frac{(3x+y)(4x-y)+(-4x+y)}{4x^2+3xy-y^2}=\frac{(3x+y)(4x-y)-(4x-y)}{4x^2+3xy-y^2}=\frac{(4x-y)(3x+y-1)}{(4x-y)(x+y)}=\frac{(3x+y-1)}{(x+y)}$
h)	$\frac{(x+3)^2}{3(x+1)}$

i)	$\dfrac{2x\left(x^2-5\right)+\left(x^2-5\right)}{x(2x+1)-\sqrt{5}(2x+1)}=\dfrac{(2x+1)\left(x^2-5\right)}{(2x+1)\left(x-\sqrt{5}\right)}=\dfrac{(2x+1)\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)}{(2x+1)\left(x-\sqrt{5}\right)}=x+\sqrt{5}$
j)	$\dfrac{x-4+m}{x+4+m}$

Ejercicio 2

a)	$\dfrac{3y^2}{3x^2-6x}\cdot\dfrac{x^2-4}{y}$ $=\dfrac{3y^2}{3x(x-2)}\cdot\dfrac{(x+2)(x-2)}{y}$ $=\dfrac{y(x+2)}{x}$
b)	$\dfrac{2w}{w-z}$
c)	$\dfrac{y-1}{2y^2+4y+2}\cdot\dfrac{(y+1)^2}{y-1}$ $=\dfrac{y-1}{2\left(y^2+2y+1\right)}\cdot\dfrac{(y+1)^2}{y-1}$ $=\dfrac{\cancel{y-1}}{2\cancel{(y+1)^2}}\cdot\dfrac{\cancel{(y+1)^2}}{\cancel{y-1}}$ $=\dfrac{1}{2}$

d)	$\dfrac{6x(7x-2y)}{3x-1}$
e)	$\dfrac{6w+12}{9w^2+6w-24} \div \dfrac{8w-12}{15w-20}$ $=\dfrac{6w+12}{9w^2+6w-24} \cdot \dfrac{15w-20}{8w-12}$ $=\dfrac{6(w+2)}{3(3w^2+2w-8)} \cdot \dfrac{5(3w-4)}{4(2w-3)}$ $=\dfrac{6\cancel{(w+2)}}{3\cancel{(3w-4)}\cancel{(w+2)}} \cdot \dfrac{5\cancel{(3w-4)}}{4(2w-3)}$ $=\dfrac{5}{2(2w-3)}$
f)	$-\dfrac{a(a+2b)}{c(a+c)}$
g)	$\dfrac{w^4-z^4}{w^2+2wz+z^2} \div \dfrac{w-z}{w^2+wz}$ $=\dfrac{w^4-z^4}{w^2+2wz+z^2} \cdot \dfrac{w^2+wz}{w-z}$ $=\dfrac{(w^2+z^2)\cancel{(w+z)}\cancel{(w-z)}}{\cancel{(w+z)^2}} \cdot \dfrac{w\cancel{(w+z)}}{\cancel{w-z}}$ $=w(w^2+z^2)$
h)	$\dfrac{(x+2)(x-4)}{x^2+2x+4}$

i)	$\frac{3a^2+11a+6}{4a^2+16a+7}\div\frac{3a^2-a-2}{2a^2-a-28}$ $=\frac{3a^2+11a+6}{4a^2+16a+7}\cdot\frac{2a^2-a-28}{3a^2-a-2}$ $=\frac{\cancel{(3a+2)}(a+3)}{\cancel{(2a+7)}(2a+1)}\cdot\frac{(a-4)\cancel{(2a+7)}}{(a-1)\cancel{(3a+2)}}$ $=\frac{(a+3)(a-4)}{(2a+1)(a-1)}$
j)	$\frac{y+3}{6(y+1)}$
k)	$\frac{3}{y^2-4}+\frac{5}{y^2-4y-12}$ $=\frac{3}{(y+2)(y-2)}+\frac{5}{(y-6)(y+2)}$ $=\frac{3(y-6)+5(y-2)}{(y+2)(y-2)(y-6)}$ $=\frac{3y-18+5y-10}{(y+2)(y-2)(y-6)}$ $=\frac{8y-28}{(y+2)(y-2)(y-6)}$
l)	$\frac{3(2x+3)}{(x-3)(x+3)}$

m)	$\frac{1}{y}-\frac{y+2}{y^2}+\frac{3}{y^3}$ $=\frac{y^2-y(y+2)+3}{y^3}$ $=\frac{y^2-y^2-2y+3}{y^3}$ $=\frac{-2y+3}{y^3}$
n)	$\frac{3(3w+4)}{(w-4)(w+4)}$
o)	$\frac{x}{(x^2+x-6)}+\frac{x}{x^2-2x}$ $=\frac{x}{(x-2)(x+3)}+\frac{x}{x(x-2)}$ $=\frac{x^2+x(x+3)}{x(x-2)(x+3)}$ $=\frac{x^2+x^2+3x}{x(x-2)(x+3)}$ $=\frac{2x^2+3x}{x(x-2)(x+3)}$ $=\frac{\cancel{x}(2x+3)}{\cancel{x}(x-2)(x+3)}$
p)	$\frac{(a+2)(a^2-a+1)}{a^2(a-1)^2}$

q)

$$\frac{8}{(y+2)^2(y+1)}-\frac{24}{(y-1)^2(y+2)}$$

$$=\frac{8(y+1)-24(y+2)}{(y+2)^2(y+1)^2}$$

$$=\frac{8y+8-24y-48}{(y+2)^2(y+1)^2}$$

$$=\frac{-16y-40}{(y+2)^2(y+1)^2}$$

$$=\frac{-8(2y+5)}{(y+2)^2(y+1)^2}$$

r)

$$-\frac{x^2-4x-3}{x(x-1)(x+1)}$$

s)

$$\left(\frac{5}{x-3}-\frac{4x+19}{x^2-9}\right)\div\frac{7x-28}{-x^3-8x^2-15x}\cdot\frac{14}{x^2+2x-15}$$

$$=\left(\frac{5}{x-3}-\frac{4x+19}{(x+3)(x-3)}\right)\div\frac{7(x-4)}{-x(x^2+8x+15)}\cdot\frac{14}{(x-3)(x+5)}$$

$$=\left(\frac{5(x+3)-(4x+19)}{(x+3)(x-3)}\right)\div\frac{7(x-4)}{-x(x+3)(x+5)}\cdot\frac{14}{(x-3)(x+5)}$$

$$=\frac{5x+15-4x-19}{(x+3)(x-3)}\cdot\frac{-x(x+3)(x+5)}{7(x-4)}\cdot\frac{14}{(x-3)(x+5)}$$

$$=\frac{\cancel{x-4}}{\cancel{(x+3)}(x-3)}\cdot\frac{-x\cancel{(x+3)}\cancel{(x+5)}}{\cancel{7}\cancel{(x-4)}}\cdot\frac{\cancel{14}}{(x-3)\cancel{(x+5)}}$$

$$=\frac{-2x}{(x-3)^2}$$

t)	$\frac{1}{2}$
u)	$\frac{y}{4(x-y)}-\frac{x}{x^2-y^2}-\left(\frac{y}{x^2-y^2}\div\frac{12}{5x+y}\right)$ $=\frac{y}{4(x-y)}-\frac{x}{(x+y)(x-y)}-\left(\frac{y}{(x+y)(x-y)}\cdot\frac{5x+y}{12}\right)$ $=\frac{y}{4(x-y)}-\frac{x}{(x+y)(x-y)}-\frac{y(5x+y)}{12(x+y)(x-y)}$ $=\frac{3y(x+y)-12x-y(5x+y)}{12(x+y)(x-y)}$ $=\frac{3xy+3y^2-12x-5xy-y^2}{12(x+y)(x-y)}$ $=\frac{-2xy+2y^2-12x}{12(x+y)(x-y)}$ $=\frac{2(-xy+y^2-6x)}{12(x+y)(x-y)}$ $=\frac{y^2-xy-6x}{6(x+y)(x-y)}$

Ejercicio 3

a)	$\dfrac{1+\dfrac{1}{a}}{1-\dfrac{1}{a}}$ $=\left(1+\dfrac{1}{a}\right)\div\left(1-\dfrac{1}{a}\right)$ $=\dfrac{a+1}{a}\div\dfrac{a-1}{a}$ $=\dfrac{a+1}{a}\cdot\dfrac{a}{a-1}$ $=\dfrac{a+1}{a-1}$
b)	$x-y$

c)	$\dfrac{\dfrac{x^2}{y}-\dfrac{x^2-y^2}{x+y}}{\dfrac{x-y}{y}+\dfrac{y}{x}}$ $=\left(\dfrac{x^2}{y}-\dfrac{x^2-y^2}{x+y}\right)\div\left(\dfrac{x-y}{y}+\dfrac{y}{x}\right)$ $=\dfrac{x^2(x+y)-y(x^2-y^2)}{y(x+y)}\div\dfrac{x(x-y)+y^2}{xy}$ $=\dfrac{x^3+x^2y-x^2y+y^3}{y(x+y)}\cdot\dfrac{xy}{x^2-xy+y^2}$ $=\dfrac{x^3+y^3}{y(x+y)}\cdot\dfrac{xy}{x^2-xy+y^2}$ $=\dfrac{\cancel{(x+y)}\,\cancel{(x^2-xy+y^2)}\,x\cancel{y}}{\cancel{y}\,\cancel{(x+y)}\,\cancel{(x^2-xy+y^2)}}$ $=x$
d)	$\dfrac{x^2(x+1)}{(x-1)^2}$

Ejercicio 4

a)	$\frac{\sqrt{a+16}-2}{a}=$ $\frac{\sqrt{a+16}-2}{a}\cdot\frac{\sqrt{a+16}+2}{\sqrt{a+16}+2}$ $=\frac{\left(\sqrt{a+16}\right)^2-(2)^2}{a\left(\sqrt{a+16}+2\right)}$ $=\frac{a+16-4}{a\left(\sqrt{a+16}+2\right)}$ $=\frac{a+12}{a\left(\sqrt{a+16}+2\right)}$
b)	$\frac{-a^2+5a+3}{\left(a^2+5\right)\left(\sqrt{5a+3}+a\right)}$
c)	$\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}}$ $=\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}}\cdot\frac{\sqrt{2-x}-\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}}$ $=\frac{\left(\sqrt{2-x}\right)^2-\left(\sqrt{2+x}\right)^2}{\left(\sqrt{2-x}-\sqrt{2+x}\right)^2}$ $=\frac{2-x-(2+x)}{\left(\sqrt{2-x}-\sqrt{2+x}\right)^2}$ $=\frac{-2x}{\left(\sqrt{2-x}-\sqrt{2+x}\right)^2}$

d)	$\dfrac{3}{\left(\sqrt{3+x^2}+x\right)^2}$
e)	$\dfrac{5-\sqrt[3]{2x}}{x+3}$ $=\dfrac{5-\sqrt[3]{2x}}{x+3}\cdot\dfrac{5^2+5\cdot\sqrt[3]{2x}+\sqrt[3]{(2x)^2}}{5^2+5\cdot\sqrt[3]{2x}+\sqrt[3]{(2x)^2}}$ $=\dfrac{125-2x}{(x+3)\left(25+5\sqrt[3]{2x}+\sqrt[3]{4x^2}\right)}$
f)	$\dfrac{2a+7}{(a^3+1)\left(\sqrt[3]{(2a+15)^2}+2\sqrt[3]{2a+15}+4\right)}$
g)	$\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{3x}$ $=\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{3x}\cdot\dfrac{\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2}}{3x\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2}\right)}$ $=\dfrac{1+x-(1-x)}{3x\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}\right)}$ $=\dfrac{2x}{3x\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}\right)}$ $=\dfrac{2}{3\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}\right)}$

h)	$\dfrac{2x+73}{\sqrt[3]{x}\left(\sqrt[3]{(2x+81)^2}+2\sqrt[3]{(2x+81)}+4\right)}$

Ejercicio 5

a)	$\dfrac{2}{\sqrt{m}-\sqrt{m+2}}=\dfrac{2}{\sqrt{m}-\sqrt{m+2}}\cdot\dfrac{\sqrt{m}+\sqrt{m+2}}{\sqrt{m}+\sqrt{m+2}}=\dfrac{2\left(\sqrt{m}+\sqrt{m+2}\right)}{m-(m+2)}=-\left(\sqrt{m}+\sqrt{m+2}\right)$
b)	$\dfrac{6x\left(\sqrt{x}-7\right)}{x-49}$
c)	$\dfrac{2-3x}{\sqrt[3]{x}-3}=\dfrac{2-3x}{\sqrt[3]{x}-3}\cdot\dfrac{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}{\sqrt[3]{x^2}+3\sqrt[3]{x}+9}=\dfrac{(2-3x)\left(\sqrt[3]{x^2}+3\sqrt[3]{x}+9\right)}{x-27}$
d)	$\dfrac{(2x+3)\left(1+\sqrt[3]{x+4}+\sqrt[3]{(x+4)^2}\right)}{-x-3}$
e)	$\dfrac{x^2\left(\sqrt[3]{(x+1)^2}+\sqrt[3]{2x+2}+\sqrt[3]{4}\right)}{x-1}$
f)	$-\dfrac{\left(\sqrt{3-x}-\sqrt{3+x}\right)^2}{2x}$

Ejercicios de autoevaluación

Ejercicio 1

a)	$m(1+m)^{\frac{-1}{2}}+2(1+m)^{\frac{1}{2}}$ $=\dfrac{m}{(1+m)^{\frac{1}{2}}}+2(1+m)^{\frac{1}{2}}$ $=\dfrac{m+2(1+m)^{\frac{1}{2}}(1+m)^{\frac{1}{2}}}{(1+m)^{\frac{1}{2}}}$ $=\dfrac{m+2(1+m)}{(1+m)^{\frac{1}{2}}}$ $=\dfrac{m+2+2m}{(1+m)^{\frac{1}{2}}}$ $\dfrac{3m+2}{(1+m)^{\frac{1}{2}}}.$
b)	$\dfrac{(4-x^2)^{\frac{1}{2}}+x^2(4-x^2)^{-\frac{1}{2}}}{4-x^2}$

	$=\dfrac{\left(4-x^2\right)^{\frac{1}{2}}+\dfrac{x^2}{\left(4-x^2\right)^{\frac{1}{2}}}}{4-x^2}$ $=\dfrac{\dfrac{4-x^2+x^2}{\left(4-x^2\right)^{\frac{1}{2}}}}{4-x^2}$ $=\dfrac{\dfrac{4}{\left(4-x^2\right)^{\frac{1}{2}}}}{4-x^2}$ $=\dfrac{4}{\left(4-x^2\right)^{\frac{1}{2}}\left(4-x^2\right)}$ $=\dfrac{4}{\left(4-x^2\right)^{\frac{3}{2}}}.$
c)	$\left(5\sqrt{\dfrac{x}{4}}-\dfrac{1}{2}\sqrt{x^3y}+6x\sqrt[4]{\dfrac{x^2y^2}{16}}-3\right)+\left(4\sqrt{4x}+x\sqrt{xy}-9\sqrt{x}\right)$ $\left(5\sqrt{\dfrac{x}{4}}-\dfrac{1}{2}\sqrt{x^3y}+6x\sqrt[4]{\dfrac{x^2y^2}{16}}-3\right)+\left(4\sqrt{4x}+x\sqrt{xy}-9\sqrt{x}\right)$ $=\left(\dfrac{5}{2}\sqrt{x}-\dfrac{1}{2}x\sqrt{xy}+\dfrac{6}{2}x\sqrt[4]{x^2y^2}-3\right)+\left(4\sqrt{2^2x}+x\sqrt{xy}-9\sqrt{x}\right)$ $=\left(\dfrac{5}{2}\sqrt{x}-\dfrac{1}{2}x\sqrt{xy}+\dfrac{6}{2}x\sqrt{xy}-3\right)+\left(4\cdot 2\sqrt{x}+x\sqrt{xy}-9\sqrt{x}\right)$ $=\dfrac{5}{2}\sqrt{x}-\dfrac{1}{2}x\sqrt{xy}+\dfrac{6}{2}x\sqrt{xy}-3+8\sqrt{x}+x\sqrt{xy}-9\sqrt{x}$ $=\dfrac{7}{2}x\sqrt{xy}+\dfrac{3}{2}\sqrt{x}-3.$
d)	$\left(2m^4-2m^3+2m-1\right)\div\left(m+\dfrac{1}{2}\right)$

Por división sintética, se tiene que:

$$\begin{array}{cccccc} 2 & -2 & 0 & 2 & -1 & \big| -\frac{1}{2} \\ \downarrow & -1 & \frac{3}{2} & -\frac{3}{4} & -\frac{5}{8} & \\ \hline 2 & -3 & \frac{3}{2} & \frac{5}{4} & -\frac{13}{8} & \end{array}$$

Así, el cociente es $2m^3-3m^2+\frac{3m}{2}+\frac{5}{4}$; y el residuo, $-\frac{13}{8}$.

e)

$$\left[\frac{2x-3}{x^2+8x+7}-\frac{x-2}{(x+1)^2}\right]\div\frac{x^3-6x^2+11x}{x^3+x^2-49x-49}$$

$$=\left[\frac{2x-3}{x^2+8x+7}-\frac{x-2}{(x+1)^2}\right]\cdot\frac{x^3+x^2-49x-49}{x^3-6x^2+11x}$$

$$=\left[\frac{2x-3}{(x+1)(x+7)}-\frac{x-2}{(x+1)^2}\right]\cdot\frac{x^3-49x+x^2-49}{x(x^2-6x+11)}$$

$$=\left[\frac{(2x-3)(x+1)-(x-2)(x+7)}{(x+1)^2(x+7)}\right]\cdot\frac{x(x^2-49)+(x^2-49)}{x(x^2-6x+11)}$$

$$=\left[\frac{2x^2+2x-3x-3-(x^2+7x-2x-14)}{(x+1)^2(x+7)}\right]\cdot\frac{(x+7)(x-7)(x+1)}{x(x^2-6x+11)}$$

$$=\left[\frac{2x^2+2x-3x-3-x^2-7x+2x+14}{(x+1)^2(x+7)}\right]\cdot\frac{(x+7)(x-7)(x+1)}{x(x^2-6x+11)}$$

$$=\left[\frac{\cancel{x^2-6x+11}}{\cancel{(x+1)}^2\,\cancel{(x+7)}}\right]\cdot\frac{\cancel{(x+7)}(x-7)\cancel{(x+1)}}{x\cancel{(x^2-6x+11)}}$$

$$=\frac{x-7}{x(x+1)}.$$

f)

$$\frac{x^5+x^3-8x^2y^3-8y^3}{x^3-2x^2+x-2}\cdot\frac{3x+9}{x^3+2x^2y+3x^2+4xy^2+6xy+12y^2}$$

$$=\frac{x^3\left(x^2+1\right)-8y^3\left(x^2+1\right)}{x^2\left(x-2\right)+x-2}\cdot\frac{3\left(x+3\right)}{x^3+3x^2+4xy^2+12y^2+2x^2y+6xy}$$

$$=\frac{\left(x^2+1\right)\left(x^3-8y^3\right)}{\left(x-2\right)\left(x^2+1\right)}\cdot\frac{3\left(x+3\right)}{x^2\left(x+3\right)+4y^2\left(x+3\right)+2xy\left(x+3\right)}$$

$$=\frac{\left(x^2+1\right)\left(x^3-8y^3\right)}{\left(x-2\right)\left(x^2+1\right)}\cdot\frac{3\cancel{\left(x+3\right)}}{\cancel{\left(x+3\right)}\left(x^2+4y^2+2xy\right)}$$

$$=\frac{\cancel{\left(x^2+1\right)}\left(x^3-8y^3\right)}{\left(x-2\right)\cancel{\left(x^2+1\right)}}\cdot\frac{3}{\left(x^2+4y^2+2xy\right)}$$

$$=\frac{\left(x-2y\right)\cancel{\left(x^2+2xy+4y^2\right)}}{x-2}\cdot\frac{3}{\cancel{\left(x^2+2xy+4y^2\right)}}$$

$$=\frac{3\left(x-2y\right)}{x-2}.$$

Ejercicio 2

a)

Si $u=x^2$**, entonces** $-4x^4+\frac{68}{3}x^2-\frac{40}{3}=-4u^2+\frac{68}{3}u-\frac{40}{3}$**. Luego,**

$\Delta=\left(\frac{68}{3}\right)^2-4\cdot-4\cdot-\frac{40}{3}=\frac{2704}{9}$**, y** $x_1=\frac{-\frac{68}{3}+\sqrt{\frac{2704}{9}}}{2\cdot-4}=\frac{2}{3}$,

$x_2=\frac{-\frac{68}{3}-\sqrt{\frac{2704}{9}}}{2\cdot-4}=5$**.**

	$-4x^4+\frac{68}{3}x^2-\frac{40}{3}$ $=-4u^2+\frac{68}{3}u-\frac{40}{3}$ $=-4\left(u-\frac{2}{3}\right)(u-5)$ $=-\frac{4}{3}(3u-2)(u-5)$ $=-\frac{4}{3}(3x^2-2)(x^2-5).$
b)	$3y+45y^2-18=3(5y-3)(3y+2)$
c)	$\frac{m^2}{2}-11m+28$ $\frac{m^2}{2}-11m+28=\frac{1}{2}(m^2-22m+56)$, por lo cual $\Delta=(-22)^2-4\cdot1\cdot56=260$ y $x_1=\frac{--22+\sqrt{260}}{2\cdot1}=11+\sqrt{65}$, $x_2=\frac{--22-\sqrt{260}}{2\cdot1}=11-\sqrt{65}$ $\frac{m^2}{2}-11m+28=\frac{1}{2}(m^2-22m+56)=\frac{1}{2}\left(m+11+\sqrt{65}\right)\left(m+11-\sqrt{65}\right)$
d)	$6x^4+4x^3-3x^2-2x$ $=2x^3(3x+2)-x(3x+2)$ $=(3x+2)(2x^3-x)$ $=(3x+2)x(2x^2-1)$ $=x(3x+2)\left(\sqrt{2}x+1\right)\left(\sqrt{2}x-1\right).$

Fuentes consultadas

Ávila, J. (2003). *Álgebra y Trigonometría*. Cartago: Editorial Tecnológica.

Barrantes, H. (2005). *Introducción a la Matemática*. San José, Costa Rica: EUNED.

Barrantes, H. (2010). *Matemática Básica para Administración*. San José, Costa Rica: EUNED.

Chacel, R. (s. f.). George Polya. Estrategias para la resolución de problemas. Recuperado de http://ficus.pntic.mec.es/fheb0005/Hojas_varias/Material_de_apoyo/-Estrategias%20de%20Polya.pdf

Goodman, A. y Hirsch, L. (1996). *Álgebra y trigonometría con geometría analítica*. México: Prentice Hall.

Larson, R. y Hostetler, R. (2010). *Precálculo* (7.ª ed.). México, D. F.: Editorial Reverté S. A.

Ministerio de Hacienda (2017). Impuesto sobre la renta (régimen tradicional). Recuperado de http://www.hacienda.go.cr/contenido/12994-regimen-tradicional

Murillo, M., Soto, A. y Araya, J. (2003). *Matemática básica con aplicaciones*. San José, Costa Rica: EUNED.

Paul, R. y Haeussler, E. (2003). *Matemáticas para administración y economía* (10.ª ed.). México: Pearson.

Ruiz, A. (2003). *Historia y Filosofía de las matemáticas*. San José, Costa Rica: EUNED.

Sullivan, M. (2013). *Álgebra y Trigonometría* (9.ª ed.). México, D. F.: Pearson.

Zill, D. y Wright, W. (2011). *Cálculo. Trascendentes tempranas* (4.ª ed.). México, D. F.: McGraw-Hill.